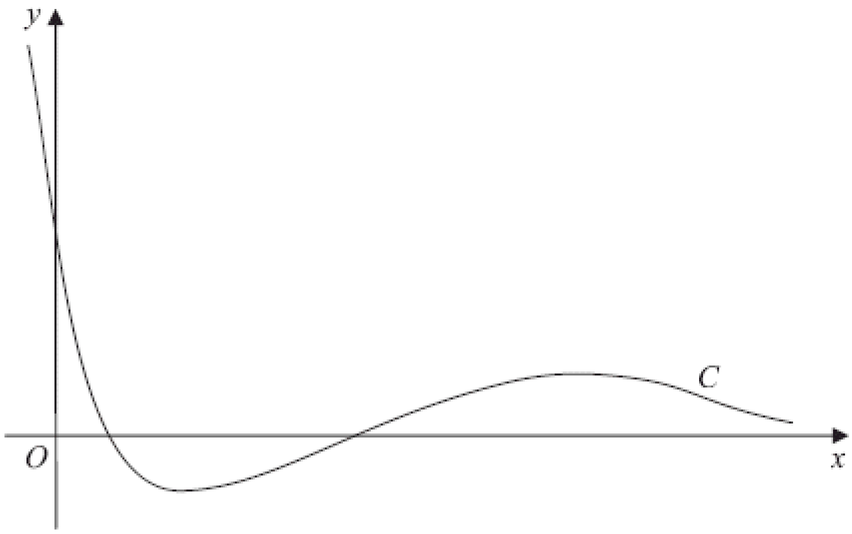
1.



The diagram above shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

(a) Find the coordinates of the point where C crosses the y-axis.

(1)

(b) Show that C crosses the x-axis at $x = 2$ and find the x-coordinate of the other point where C crosses the x-axis.

(3)

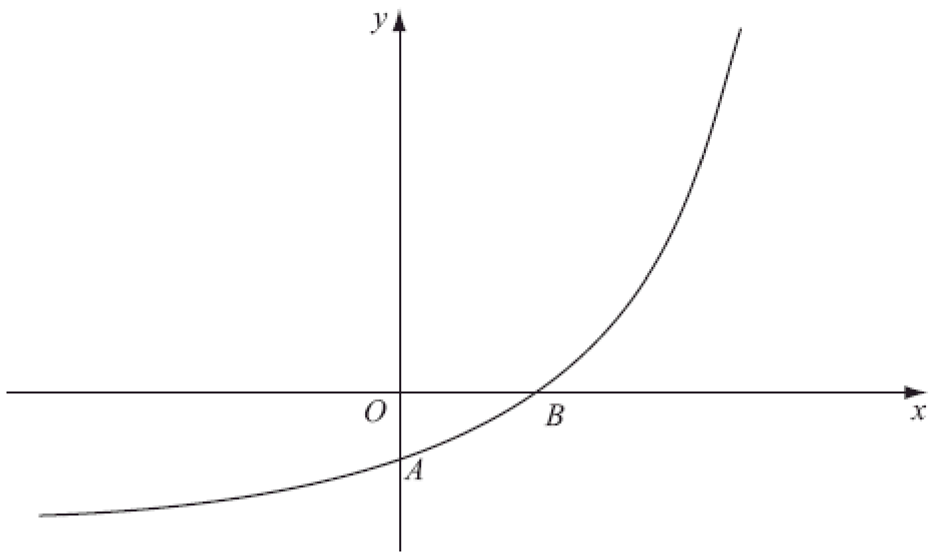
(c) Find $\frac{dy}{dx}$.

(3)

(d) Hence find the exact coordinates of the turning points of C.

(5)
(Total 12 marks)

2.



The figure above shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$

The curve meets the coordinate axes at the points $A(0, 1 - k)$ and 1 $B\left(\frac{1}{2}\ln k, 0\right)$, where k is a constant and $k > 1$, as shown in the diagram above.

On separate diagrams, sketch the curve with equation

(a) $y = |\mathrm{f}(x)|$,

(3)

(b) $y = \mathrm{f}^{-1}(x)$

(2)

Show on each sketch the coordinates, in terms of k, of each point at which the curve meets or cuts the axes.

Given that $\mathrm{f}(x) = \mathrm{e}^{2x} - k$,

(c) state the range of f,

(1)

(d) find $\mathrm{f}^{-1}(x)$,

(3)

(e) write down the domain of f^{-1}.

(1)
(Total 10 marks)

3. The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$. The x-coordinate of P is 3.

Find an equation of the normal to the curve at the point P in the form $y = ax + b$, where a and b are constants.

(Total 5 marks)

4. Every £1 of money invested in a savings scheme continuously gains interest at a rate of 4% per year. Hence, after x years, the total value of an initial £1 investment is £y, where

$$y = 1.04^x.$$

(a) Sketch the graph of $y = 1.04^x$, $x \geq 0$.

(2)

(b) Calculate, to the nearest £, the total value of an initial £800 investment after 10 years.

(2)

(c) Use logarithms to find the number of years it takes to double the total value of any initial investment.

(3)
(Total 7 marks)

5. The curve with equation $y = \ln 3x$ crosses the x-axis at the point P $(p, 0)$.

(a) Sketch the graph of $y = \ln 3x$, showing the exact value of p.

(2)

The normal to the curve at the point Q, with x-coordinate q, passes through the origin.

(b) Show that $x = q$ is a solution of the equation $x^2 + \ln 3x = 0$.

(4)

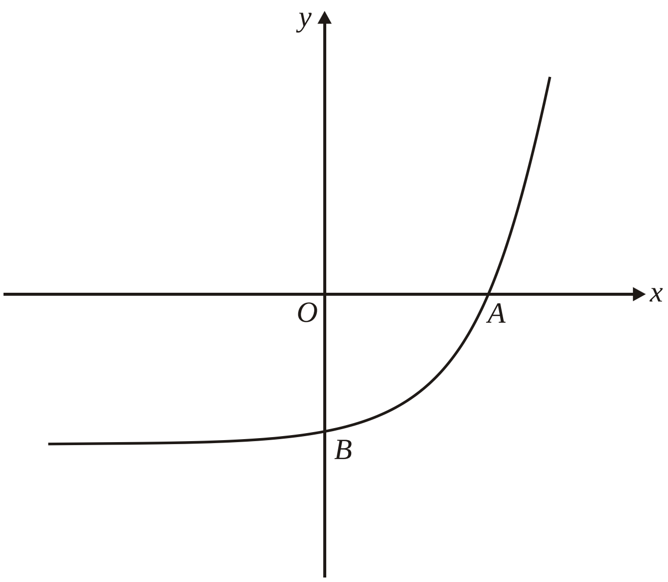
(c) Show that the equation in part (b) can be rearranged in the form $x = \frac{1}{3}e^{-x^2}$.

(2)

(d) Use the iteration formula $x_{n+1} = \frac{1}{3}e^{-x_n^2}$, with $x_0 = \frac{1}{3}$, to find x_1, x_2, x_3 and x_4. Hence write down, to 3 decimal places, an approximation for q.

(3)
(Total 11 marks)

6.



The diagram above shows a sketch of the curve with equation $y = f(x)$ where the function f is given by

$$f: x \mapsto e^{x-2} - 1, \quad x \in \mathbb{R}.$$

The curve meets the x-axis at the point A and the y-axis at the point B.

(a) Write down the coordinates of A and B.

(2)

(b) Find, in the form $f^{-1}(x): x \mapsto \ldots$, the inverse function of f and state its domain.

(5)

(c) Prove that the equation $f(x) = x$ has a root α in the interval [3, 4].

(2)

(d) Use the iterative formula

$$x_{n+1} = f^{-1}(x_n), \quad \text{with } x_1 = 3.5,$$

to find α to 3 decimal places. Prove that your answer is correct to 3 decimal places.

(5)

(Total 14 marks)

7. (a) Sketch the curve with equation $y = \ln x$.

(2)

(b) Show that the tangent to the curve with equation $y = \ln x$ at the point (e, 1) passes through the origin.

(3)

(c) Use your sketch to explain why the line $y = mx$ cuts the curve $y = \ln x$ between $x = 1$ and $x = \text{e}$ if $0 < m < \dfrac{1}{\text{e}}$.

(2)

Taking $x_0 = 1.86$ and using the iteration $x_{n+1} = \text{e}^{\frac{1}{3}x_n}$,

(d) calculate x_1, x_2, x_3, x_4 and x_5, giving your answer to x_5 to 3 decimal places.

(3)

The root of $\ln x - \frac{1}{3}x = 0$ is α.

(e) By considering the change of sign of $\ln x - \frac{1}{3}x$ over a suitable interval, show that your answer for x_5 is an accurate estimate of α, correct to 3 decimal places.

(3)

(Total 13 marks)

1.	(a)	Either $y = 2$ or $(0, 2)$	B1	1
	(b)	When $x = 2,\ y = (8 - 10 + 2)e^{-2} = 0e^{-2} = 0$	B1	
		$(2x^2 - 5x + 2) = 0 \Rightarrow (x - 2)(2x - 1) = 0$		
		Either $x = 2$ (for possibly B1 above) or $\underline{x = \frac{1}{2}}$.	A1	3

Note

If the candidate believes that $e^{-x} = 0$ solves to $x = 0$ or gives an extra solution of $x = 0$, then withhold the final accuracy mark.

(c)	$\frac{dy}{dx} = (4x - 5)e^{-x} - (2x^2 - 5x + 2)e^{-x}$	A1 A1	3

Note

(their u')e^{-x} + $(2x^2 - 5x + 2)$(their v')
A1: Any one term correct.
A1: Both terms correct.

(d)	$(4x - 5)e^{-x} - (2x^2 - 5x + 2)e^{-x} = 0$			
	$2x^2 - 9x + 7 = 0 \Rightarrow (2x - 7)(x - 1) = 0$			
	$x = \frac{7}{2}, 1$		A1	
	When $x = \frac{7}{2},\ y = 9e^{-\frac{7}{2}}$, when $x = 1,\ y = -e^{-1}$	dd	A1	5

Note

1st For setting their $\frac{dy}{dx}$ found in part (c) equal to 0.

2nd Factorise or eliminate out e^{-x} correctly and an attempt to factorise a 3-term quadratic or apply the formula to candidate's $ax^2 + bx + c$.
See rules for solving a three term quadratic equation on page 1 of this Appendix.

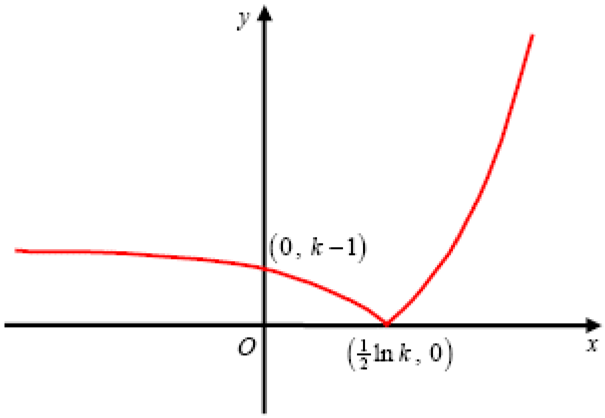
3rd ddM1: An attempt to use at least one x-coordinate on $y = (2x^2 - 5x + 2)e^{-x}$.
Note that this method mark is dependent on the award of the two previous method marks in this part.
Some candidates write down corresponding y-coordinates without any working. It may be necessary on some occasions to use your calculator to check that at least one of the two y-coordinates found is correct to awrt 2 sf.

Final A1: Both$\{x = 1\}$, $y = -e^{-1}$ and $\{x = \frac{7}{2}\}$, $y = 9e^{-\frac{7}{2}}$. **cao**

Note that both exact values of y are required.

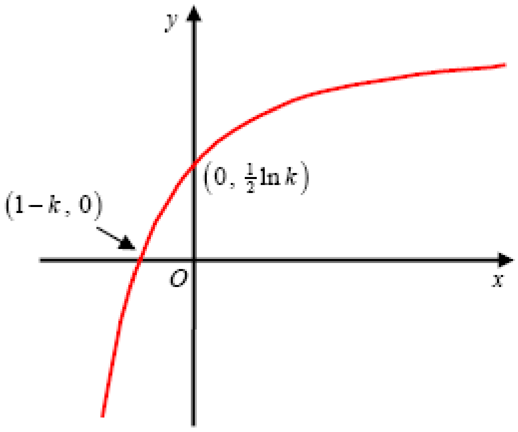
[12]

2. (a)



Curve retains shape when $x > \frac{1}{2}\ln k$		B1	
Curve reflects through the x-axis when $x > \frac{1}{2}\ln k$		B1	
(0, k – 1) and ($\frac{1}{2}\ln k$,0) marked in the correct positions.		B1	3

(b)



Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote)		B1	
(1 – k, 0) and (0, $\frac{1}{2}\ln k$)		B1	2

(c) Range of f: f(x) > –k or y > –k or (–k, ∞)	Either f(x) > –k or y> –k or (– k, ∞)or f > – k or Range > – k .	B1	1

(d) $y = e^{2x} - k \Rightarrow y + k = e^{2x}$ — Attempt to make x (or swapped y) the subject

$\Rightarrow \ln(y + k) = 2x$

$\Rightarrow \frac{1}{2}\ln(y+k) = x$ — Makes e^{2x} the subject and takes ln of both sides

Hence $f^{-1}(x) = \underline{\frac{1}{2}\ln(x+k)}$ — $\underline{\frac{1}{2}\ln(x+k)}$ or $\underline{\ln\sqrt{(x+k)}}$ — A1 cao — 3

(e) $f^{-1}(x)$: Domain: $\underline{x > -k}$ or $\underline{(-k,\infty)}$ — Either $\underline{x > -k}$ or $\underline{(-k,\infty)}$ or Domain $> -k$ or x "ft one sided inequality" their part (c) RANGE answer — B1ft — 1

[10]

3. $\frac{dy}{dx} = \frac{1}{x}$ — A1

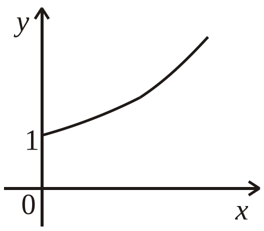
At $x = 3$, gradient of normal $= \dfrac{-1}{\frac{1}{3}} = -3$

$y - \ln 1 = -3(x - 3)$

$y = -3x + 9$ — A1 — 5

[5]

4. (a)



Shape — B1

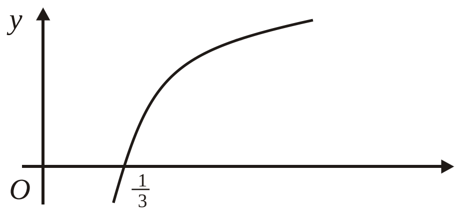
domain, intercept — B1 — 2

(b) £800 × 1.04^{10} ≈ £1184 — A1 — 2

cao

	(c)	$1.04^x = 2$		
		$x = \dfrac{\ln 2}{\ln 1.04} \approx 18$ (years)	A1	3
		accept 17.7, 17 years 8 months		

[7]

5.	(a)			
		Shape	B1	
		$p = \frac{1}{3}$ or $\{\frac{1}{3}, 0\}$ seen	B1	2
	(b)	Gradient of tangent at $Q = \dfrac{1}{q}$	B1	
		Gradient of normal = $-q$		
		Attempt at equation of OQ [$y = -qx$] and substituting $x = q$, $y = \ln 3q$		
		or attempt at equation of tangent [$y - 3\ln q = -q(x - q)$] with $x = 0, y = 0$		
		or equating gradient of normal to $(\ln 3q)/q$		
		$q^2 + \ln 3q = 0$ (*)	A1	4
	(c)	$\ln 3x = -x^2 \Rightarrow 3x = e^{-x^2}$; $\Rightarrow x = \frac{1}{3}e^{-x^2}$	A1	2
	(d)	$x_1 = 0.298280$; $x_2 = 0.304957$, $x_3 = 0.303731$, $x_4 = 0.303958$	A1	
		Root = 0.304 (3 decimal places)	A1	3

[11]

6.	(a)	A is (2, 0); B is (0, $e^{-2} - 1$)	B1; B1	2

(b) $y = e^{x-2} - 1$

Change over x and y, $x = e^{y-2} - 1$

$y - 2 = \ln(x + 1)$

$y = 2 + \ln(x + 1)$ A1

$f^{-1} : x / 2 + \ln(x + 1), x > -1$ A1 A1 5

(c) $f(x) - x = 0$ is equivalent to $e^{x-2} - 1 - x = 0$

Let $g(x) = e^{x-2} - 1 - x$

$g(3) = -1.28\ldots$

$g(4) = 2.38\ldots$

Sign change $\Rightarrow$ root α A1 2

(d) $x_{n+1} = 2 + \ln(x_n + 1)$, $x_1 = 3.5$

$x_2 = 3.5040774$ A1

$x_3 = 3.5049831$ A1

$x_4 = 3.5051841$

$x_5 = 3.5052288$

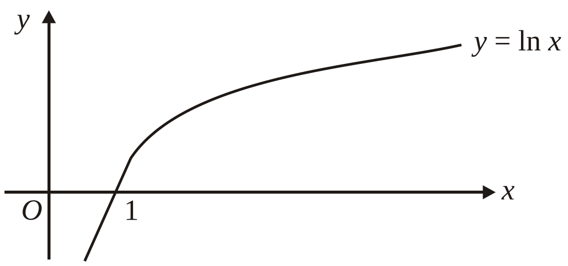
Needs convincing argument on 3 d.p. accuracy

Take 3.5053 and next iteration is reducing 3.50525…

Answer: 3.505 (3 d.p.) A1 5

[14]

7. (a)



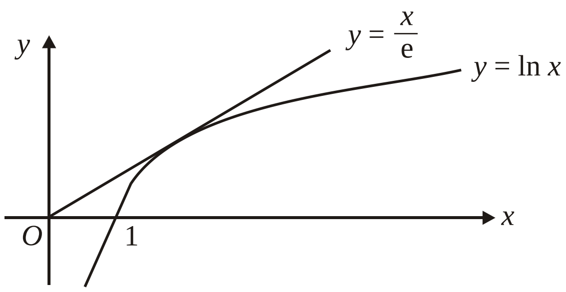
B1 shape

B1 x-intercept labelled 2

(b) $\frac{dy}{dx} = \frac{1}{x}$ so tangent line to (e, 1) is $y = \frac{1}{e}x + C$

the line passes through (e, 1) so $1 = e\frac{1}{e} + C$ and $C = 0$

The line passes through the origin. A1 3



(c) All lines $y = mx$ passing through the origin and having a gradient > 0 lie above the x-axis.

Those having a gradient < $\frac{1}{e}$ will lie below the line. B1

$y = \frac{x}{e}$ so it cuts $y = \ln x$ between $x = 1$ and $x = e$. B1 2

(d) $x_0 = 1.86$

$x_1 = e^{\frac{x_n}{3}} = 1.859$

$x_2 = 1.858$ A1

$x_3 = 1.858$

$x_4 = 1.858$

$x_5 = 1.857$ A1 3

(e) When $x = 1.8575$, $\ln x - \frac{1}{3}x = 0.000\,064\,8\ldots > 0$

When $x = 1.8565$, $\ln x = -0.000\,140\ldots < 0$ A1

Change of sign implies there is a root between. A1 3

[13]

1. This question was extremely well answered with 84% of candidates gaining at least 7 of the 12 marks available and about 42% gaining all 12 marks.

Nearly all candidates were successful in answering part (a). A few candidates were initially confused when attempting part (a) by believing that the curve met the y-axis when $y = 0$. These candidates quickly recovered and relabelled part (a) as their part (b) and then went onto to find in part (a) that when $x = 0$, $y = 2$. Therefore, for these candidates, part (b) was completed before part (a).

In part (b), some candidates chose to substitute $x = 2$ into $y = (2x^2 - 5x + 2)e^{-x}$ in order to confirm that $y = 0$ The majority of candidates, however, set $y = 0$ and solved the resulting equation to give both $x = 2$ and $x = 0.5$ Only a few candidates wrote that $x = 0$ is a solution of $e^{-x} = 0$.

In part (c), the product rule was applied correctly to $(2x^2 - 5x + 2)e^{-x}$ by a very high proportion of candidates with some simplifying the result to give $(-2x^2 + 9x - 7)e^{-x}$ Common errors included either e^{-x} being differentiated incorrectly to give e^{-x} or poor bracketing. The quotient rule was rarely seen, but when it was it was usually applied correctly.

In part (d), the majority of candidates set their $\frac{dy}{dx}$ in part (c) equal to 0, although a few differentiated again and set $\frac{d^2y}{dx^2} = 0$. At this stage, few candidates produced invalid logarithmic work and lost a number of marks. Some other candidates made bracketing and/or algebraic errors in simplifying their gradient function. Most candidates realised that they needed to factorise out e^{-x} and solve the resulting quadratic with many of them correctly finding both sets of coordinates. Some candidates did not give their y-coordinates in terms of e, but instead wrote the decimal equivalent.

2. In part (a), almost all candidates realised that the transformed curve was the same as the original curve for $x > \frac{1}{2}$ ln k, and only a few failed to reflect the other part of the curve correctly through the x-axis. A significant number of candidates in this part struggled to write down the correct coordinates for the y-intercept in terms of k. The most common incorrect answers were (0, k + 1)and (0, 1 – k). A few candidates did not state the coordinates of the y-intercept in the simplest form. An answer of $(0, |1 - k|)$ was accepted on the y-axis in this part.

In part (b), many candidates realised that they needed to reflect the given curve through the line $y = x$. Although the majority of these candidates managed to draw the correct shape of the transformed curve, a few seemed unable to visualise the correct position after reflection and incorrectly positioned their curve going through the fourth quadrant. Those candidates who correctly reflected the original curve through the line $y = x$ were often unable to see that the effect of reflecting points A and B in the line $y = x$ was a reversal of x- and y-coordinates. The most common incorrect coordinates for the x and y intercepts in this part were $\left(-\frac{1}{2}\ln k, 0\right)$ and $(0, -1 + k)$ respectively.

In parts (a) and (b), examiners were fairly tolerant with curvature. Those candidates, however, who drew curves going back on themselves to give an upside-down U in the second quadrant in part (a) or a C-shape in the third quadrant in part (b) were not awarded the relevant mark for the shape of their curve.

The majority of candidates struggled with part (c). Some candidates sketched the curve of $y = e^{2x}$ and proceeded to translate this curve down k units in the y-direction and in most cases these candidates were able to write down the correct range. A significant number of candidates wrote their range using x rather than using either y or f (x). Common incorrect answers for the range were $y \in \mathbb{R}$, $y > 0$, $y > k$ or $y \geq k$

Part (d) was well answered and a majority of candidates were able to score some marks with a large number scoring full marks. The general procedure of changing the subject and switching x and y was well known. There was the occasional difficulty with taking logarithms of both sides. A common error was an incorrect answer of $\frac{1}{2}\ln(x - k)$. As x only appears once in the original function, a few candidates instead chose to use a flowchart method to find the inverse. Nearly all those who did this arrived at the correct answer.

Again, as with part (c), many candidates struggled to give the correct domain for the inverse function in part (e). Those candidates who correctly stated that the domain of the inverse function is the same as the range of the original function failed in many cases to obtain the follow-through mark. This was because they did not change y or f (x) in their range inequality to x in their domain inequality.

3. Pure Mathematics P2

Those candidates who could differentiate the logarithmic function had no difficulty with the question. A gradient function of the form $\frac{k}{x}$ was expected. Many candidates ran into problems with the $\frac{1}{3}$, and some obtained the correct expression $\dfrac{\frac{1}{3}}{\frac{x}{3}}$ but did not simplify this correctly.

Most candidates obtained the correct value, $y = 0$ when $x = 3$, but some had problems with $ln1$. Almost all candidates offered a sensible attempt at the equation of the normal rather than the tangent.

Core Mathematics

The differentiation proved difficult and both $\frac{3}{x}$ and $\frac{1}{3x}$ were common. Even if a correct expression for $\frac{dy}{dx}$ was found, difficulties with simplifying fractions often led to incorrect work. For example, $\frac{\frac{1}{3}}{\frac{1}{3}x}$ was sometimes seen simplified to x. Failure to read the question carefully lead a number of candidates to give the tangent rather than the normal. There were also candidates who did not find a numerical gradient and gave a non-linear equation for the normal.

4. The purpose of this sketch was to show the overall shape and orientation of the graph. Many sketches seen were indistinguishable from a straight line and some candidates had clearly been using graphic calculators without being how taught to use appropriate scale factors. Many who did produce curves failed to note that the domain was restricted to $x = 0$. Part (b) was well done although it was surprising to see at this level a substantial number who carried out ten separate multiplications by 1.04. This method did gain the two marks but was not good time management. A few calculated expressions like 832^{10} and failed to notice that £1.589×10^{29} was an unreasonable answer. In part (c) many could not handle the fact that no initial sum was specified and were unable to reduce the problem to an equation in a single variable. Those who did obtain $1.04^x = 2$ were usually able to complete the question.

5. This part of the syllabus undoubtedly causes candidates much difficulty but it was disappointing to see so many unable to sketch $y = \ln 3x$ correctly, clearly showing it hit or pass through the y-axis, or having the wrong curvature. Finding the value of p was also too often incorrect. Full marks were extremely rare in part (b), although the first two marks were often gained. A common error was to produce an equation for the normal which was non-linear. Parts (c) and (d) were more familiar and answered much better, although errors such as $x^2 + \ln 3x = 0 \Rightarrow e^{x^2} + 3x = 0$ in (c) and the misreading of e^{-x^2} as e^{+x^2} or, more likely, incorrectly using a calculator to evaluate e^{-} in part (d) were quite common.

6. No Report available for this question.

7. No Report available for this question.